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Modeling and Control of Anti-lock Braking systems considering different representations for tire-road interaction

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***Abstract*—Mobility and traffic safety is one of the major concerns in today's society as traffic accidents continue to be a leading cause of death on the planet. Traffic safety is influenced by several factors, one of the most important being vehicle control, by the driver, in all situations. Modern safety systems, among them, the anti-locking brake system (ABS) has allowed the reduction of uncontrollable incidents in traffic, protecting the passengers from accidents. Together with safety issues, the costumers growing demands when purchasing vehicles regarding efficiency, lower consumption, reduced emissions and high comfort encourages engineers to seek for better model representations of the vehicle dynamics and improved and simplified control techniques. This article approaches a modeling and simulation of an ABS system considering different theories for implementation of tire-road contact.**

***Keywords*—Anti-lock Braking Systems, system modeling, PID Control, Matlab/Simulink.**

I. Introduction

Mobility and traffic safety is one of the biggest concerns in today's society as traffic accidents remain one of the leading causes of death on the planet. According to a report from the World Health Organization, about 1.35 million people die each year from road traffic crashes [1].

Road safety is influenced by several factors, one of the most important being the driver control over the vehicle under all circumstances. In emergency situations the driver relies on the brakes to stop the vehicle, however excessive brake force can result in the wheel getting lock, which means that the wheel speed reached zero while the vehicle remains with longitudinal speed.

Modern safety systems, including the anti-lock braking system (ABS) have enabled the reduction of those uncontrollable driving incidents, avoiding accidents. The ABS system certifies the vehicle control while braking on different surfaces by keeping the coefficient of friction as close as possible to optimum, which not only helps to prevent wheel locking but also minimizes stopping distance. There are two classical control approaches used in ABS systems the first one is based in the control of the the wheel slip while braking and the second one in the control of wheel deceleration rate.

Different types of controllers can be used in those approaches, the most widely used by the industry today is the PID (Proportional-Integral-Derivative) controller [2]] . The PID controller is a classic control technique, its popularity can be attributed, in part, to its robust performance under a wide range of operating conditions and partly to its functional simplicity.

This article features a model of anti-lock braking system (ABS), considering different representations of tire-road interaction and the design of a classical controller for the model presented.

II. Dynamic Model

The braking system can be understood by analyzing the vehicle dynamics (lateral, longitudinal and vertical), and associated powertrain systems. Since most of the braking movement is in the longitudinal direction, in this article it is analyzed only that dynamic, what means that suspension system, load transfer between wheels and lateral forces are not considered.

Further considerations are made:

- vehicle wheels are considered dynamically decoupled;
- all wheel are rolling in same type of road;
- yaw moment is not considered;
- the vehicle is considered to be braking in a straight line and performing a complete brake (vehicle full stop);
- aerodynamic forces have minimum impact and can be neglected;
- the link between wheel and road is considered rigid;
- the wheel radius is constant during the braking.

In this section it is presented a mathematical development for modeling of wheel and the brake actuator. During wheel modeling it is also discussed the different approaches, most used in literature, to represent the tire-road interaction.

A. Wheel modeling

The modeling of the tire-wheel system in the longitudinal direction is obtained from the analysis of the movement of the wheel. Figure 1 shows the forces acting on a wheel of the vehicle during the braking process. It is being considered that when the brake is pressed, no motor torque is demanded (the accelerator pedal is at rest).

Analyzing figure 1, the equation of wheel motion is obtained from Newton's second law for translational and rotational motion.

From translational motion:

$$m\dot{v} = -\mu F_N \tag{1}$$

From rotational motion:

$$J\dot{\omega} = RF_x - T_b \tag{2}$$

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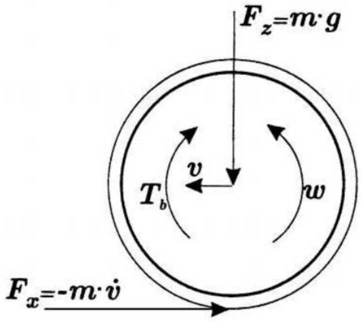

Fig. 1. Forces acting in the wheel.Figure from [4]

Where m is the vehicle mass, v is the vehicle longitudinal speed, ω is the wheel rotational speed, μ the friction coefficient, F_N the vertical force, T_b is the braking torque, J is the wheel moment of inertia, R is the wheel radius, and F_x is the frictional force $F_x = \mu F_N$.

At the tire contact with the road there are losses in transmission of longitudinal force that are dependent on the tire material and the road characteristic in which the vehicle travels [5]. Those losses results in a slippage in wheel what invalidates the relation $v = \omega R$. The slip is then given by:

For braking the vehicle:

$$\lambda = \frac{v - \omega R}{v} \tag{3}$$

Where λ is the slippage, v the longitudinal speed of the vehicle and ω the speed of the tire.

From equation 3 it is possible to see that for $\omega R = v$, the slip will be zero, which means a free-rolling wheel and for $\omega = 0$, the slip is one which means a locked wheel.

Since rotational and translational speeds are a function of the friction coefficient and the friction coefficient is a function of tire and road type, in modeling the system it is important to represent the different road types.

Each type of road has a specific characteristic curve. Consolidated models for tire-road friction coefficient modeling are the Pacejka model or magic formula and the Buckhardt model that will be explored in the next topic.

1) Pacejka Model: One of the best-known models for tire-road contact representation is the Pacejka model. The purpose of this session is not the development of the Pacejka model, but only the exposure of the developed model that was used for tire-wheel contact simulation. For more details refers to [6].

The Pacejka model is defined by the following equation:

$$Y(x) = Dsin(Catan(Bx - E(Bx - atan(Bx)))) \tag{4}$$

where,

D: determines the maximum point of the characteristic curve, being called a peak factor;

C: mainly influences the shape of the curve;

B: stiffness factor;

E: modifies around the maximum point of the curve, being, therefore, a curvature factor.

The coefficients are obtained experimentally and can be observed in table I for the different type of roads:

TABLE I
COEFFICIENTS FOR PACEJKA MODEL.

Type of roads	B	C	D	E
Dry tarmac	10	1.9	1	0.97
Wet tarmac	12	2.3	0.82	1
Snow	5	2	0.3	1
Ice	4	2	0.1	1

The curve for each road types, using Pacejka model is plotted in figure 2.

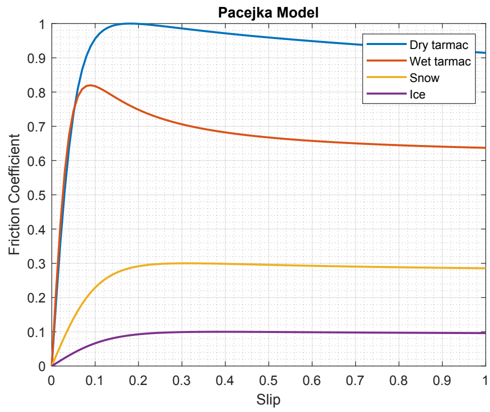

Fig. 2. Graphic of friction coefficient versus wheel slip for different road types considering Pacejka modeling. It was obtained using Matlab.

2) Modelo de Burckhardt: A second empirical model used in literature to map the relation of the friction coefficient and slip of the wheel is the Burckhardt model. It is represented by the following equation:

$$\mu = (C1(1 - e^{-C2\lambda}) - C3\lambda)e^{-C4\lambda v} \tag{5}$$

where,

C1: is the maximum value of the friction curve;

C2: mainly influences the shape of the curve;

C3: is the difference between maximum value of the curve and the value at $\lambda = 1$;

C4: curve value between $0.02 - 0.04 s/m$

The coefficients are also obtained from experiments for each specific conditions. The coefficients considered for the simulation are listed in the table II.

TABLE II
COEFFICIENTS FOR BURCKHARDT MODEL

Types of roads	B	C	D	E
Dry tarmac	1.029	17.16	0.523	0.03
Wet tarmac	1.197	29	0.0646	0.03
Snow	0.1946	94.129	0.0646	0.03
Ice	0.05	306.39	0	0.03

It can be observed that in addition to the slip itself, Burckhardt's model is dependent on the speed of the vehicle.

The curves that represent the relation of the coefficient of friction and slip, according to Burckhardt's modeling, were plotted in Matlab for a speed of $90km/h$ and are illustrated in the figure 3.

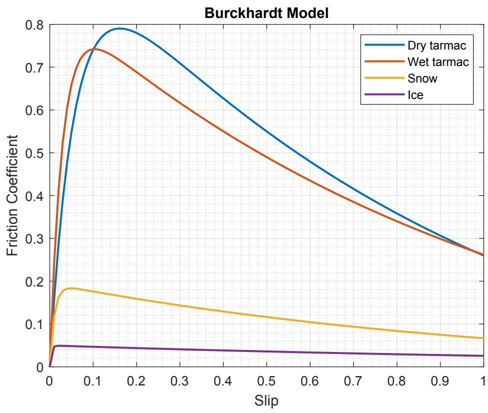

Fig. 3. Graphic of friction coefficient versus wheel slip for different road types considering Burckhardt modeling. It was obtained using Matlab.

There is a simplified version of the Burckhardt model where speed dependency is not considered. The simplified model is equated as follows:

$$\mu = c1(1 - e^{-c2\lambda}) - c3\lambda \tag{6}$$

The coefficients considered for the simulation are listed in the table III

TABLE III
COEFFICIENTS FOR SIMPLIFIED BURCKHARDT MODEL.

Type of roads	**C1**	**C2**	**C3**
Dry tarmac	1.2801	23.99	0.52;
Wet tarmac	0.857	33.822	0.3470;
Snow	0.1946	94.129	0.0646;
Ice	0.05	306.39	0;

The curves obtained by Burckhardt's simplified relation are presented in the graph of figure 4, obtained from Matlab.

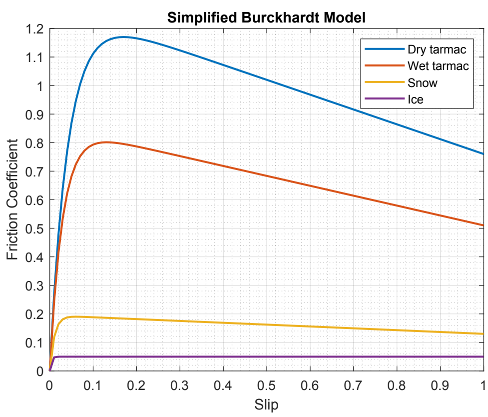

Fig. 4. Graphic of friction coefficient versus wheel slip for different road types considering Simplified Burckhardt modeling.

B. Brake actuator modeling

The most common brake actuators are mechanical-hydraulic, electro-hydraulic and electro-mechanical. The brake actuator determines how the braking force exerted by the driver on the pedal reaches the wheel, i.e. it is responsible to transform braking pressure into braking torque. An electro-Hydraulic actuator is considered in this article.

The brake actuator is modeled according to [7]. The dynamic response of a hydraulic brake system is usually modeled by a first order transfer function delayed below 0.1 seconds [8].

The relation between hydraulic line pressure and wheel brake torque is given by:

$$G_{cal} = \frac{K_{b\_cal}}{\tau_{b\_cal} * s + 1} \tag{7}$$

where τ_{cal} is the time constant of the brake caliper system and K_b is the gain.

Similarly, the hydraulic modulator can be modeled as a stable first order function:

$$G_{EHB} = \frac{1}{\tau_{EHB} * s + 1} \tag{8}$$

where τ_{EHA} is the actuator time constant.

The full modeling of the hydraulic brake is represented by:

$$G_b = G_{EHA}(s) G_{cal} \tag{9}$$

C. Nonlinear Model

The nonlinear model is obtained directly by the wheel dynamic, represented by equations 1, 2 and 3. Knowing that the idea of the ABS system is to keep the vehicle at an ideal coefficient of friction, which indicates an ideal slip value, an important equation to be developed is the slip derivative.

$$\dot{\lambda} = \frac{-R}{v}\dot{\omega} + \frac{\omega R}{v^2}\dot{v} \tag{10}$$

Substituting the traslacional and rotational accelerations in equation 10, it gets:

$$\dot{\lambda} = \frac{-1}{v}\left(\frac{(1-\lambda)}{m} + \frac{R^2}{J}\right)\mu F_N + \frac{R}{vJ}T_b \tag{11}$$

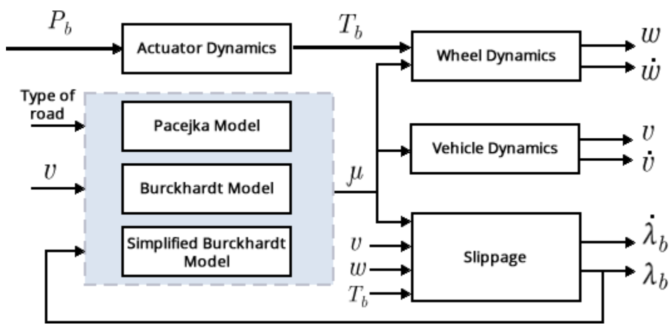

Fig. 5. Block diagram of braking system showing connections between considered modules.

In figure 5 a block diagram illustrates the connection between considered modules.

D. Linearized Model

Two approaches are considered for model linearization, the first and simpler one considers v as a slowly varying parameter compared to the rotational dynamics and the second approach considers the vehicle speed variation [9].

The first step on linearizing a system is to find the equilibrium point, where the dynamic terms are set to zero. Considering equation 10, setting $\dot{\lambda} = 0$ it is obtained the following equilibrium point:

$$T_b = \left(\left(\frac{J(1-\lambda)}{mR} + R \right) \mu F_N \right) \tag{12}$$

In order to linearize the sliding derivative equation, the approximation of μ is obtained from the Taylor series expansion. Disregarding higher order terms, one has:

$$\mu = \mu_o + \frac{\delta \mu}{\delta \lambda}|_{\lambda=\lambda_o}(\lambda - \lambda_o) \tag{13}$$

Considering $\mu_1 = \frac{\delta\mu}{\delta\lambda}|_{\lambda=\lambda_o}$, $\Delta\lambda = (\lambda - \lambda_o)$ e $\Delta T_b = T_b - T_{b_o}$, one gets the slip derivative linearization around the equilibrium point as:

$$\dot{\Delta\lambda} = \frac{F_N}{v_o} \left(\frac{\mu_o}{m} - \mu_1 \left(\frac{(1-\lambda_o)}{m} + \frac{R^2}{J} \right) \right) \Delta\lambda + \frac{R}{v_o J} \Delta T_b \tag{14}$$

By taking the Laplace transform, it is obtained the transfer function, considering the slip λ as output and brake torque T_b as input.

$$G(s) = \frac{\Delta\lambda}{\Delta T_b} = \frac{R/(v_o J)}{s + \frac{F_N}{m v_o} \left(\mu_1 \left((1-\lambda_o) + \frac{mR^2}{J} \right) - \mu_o \right)} \tag{15}$$

In order to assemble the system into state-space the desired state variables were defined, which would be v and ω.

To linearize the traslational and rotational accelerations equations, it was made the approximation of $\mu(\lambda(v,\omega))$ using the Taylor series expansion, ignoring higher order terms:

$$\mu = \mu_o + \frac{\delta\mu}{\delta\lambda}\frac{\delta\lambda}{\delta v}\Delta v + \frac{\delta\mu}{\delta\lambda}\frac{\delta\lambda}{\delta\omega}\Delta\omega \tag{16}$$

$$\mu = \mu_o + \mu_1 \frac{\omega R}{v^2}\Delta v - \mu_1 \frac{R}{v}\Delta\omega \tag{17}$$

Replacing μ in equations 1 and 2:

$$\dot{\Delta v} = \frac{-F_N}{m} \left(\mu_1 \frac{\omega_o R}{v_o^2}\Delta v - \mu_1 \frac{R}{v_o}\Delta\omega \right) \tag{18}$$

$$\dot{\Delta\omega} = \frac{1}{J} \left(F_N \mu_1 \frac{\omega_o R^2}{v_o^2}\Delta v - F_N \mu_1 \frac{R^2}{v_o}\Delta\omega - \Delta T_b \right) \tag{19}$$

The linearized system in state space reuslts in: $x_1 = \Delta v$, $x_2 = \Delta\omega$

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} -F_N \mu_1 \frac{\omega R}{v_o^2 m} & F_N \mu_1 \mu_1 \frac{R}{v_o m} \\ F_N \mu_1 \frac{\omega R^2}{v_o^2 J} & -F_N \mu_1 \frac{R^2}{v_o J} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-1}{J} \end{bmatrix} \Delta T_b \tag{20}$$

III. Control Design

In this article it was used a PID controller to keep the wheel slippage around an optimum point. The PID controller acts directly on the error signal, which is given by the difference between the measured process variable and the desired reference. Refer to [10] for further details on PID controllers.

Together with the PID control it was used a Ziegler-Nichols classical method for tuning known as modified critical gain method.

The method is based on finding the critical gain for which the system has a constant oscillation. The critical gain is found by zeroing the controller integral and derivative gain values and gradually increasing the proportional gain value.

Controller gains are obtained from the critical gain value and the oscillation period as shown in the table **??**.

TABLE IV
PID gains using modified Ziegler-Nichols critical gain method.

K_P	$\frac{K_{cr}}{3}$
K_I	$\frac{0.66K_{cr}}{T_{cr}}$
K_D	$\frac{K_{cr}T_{cr}}{9}$

IV. Simulation and Results

The system modeled was simulated using Matlab/Simulink. The vehicle parameters used for simulation are listed in table V. Note that since it was used a quarter car model to represent the vehicle dynamic, it is also considered a quarter car mass.

TABLE V
Vehicle parameters used for simulation.

Parameters	Symbol	Value	Unit
Mass	m	225	kg
Wheel radius	R	0.3	m
Gravity acceleration	g	9.80665	m/s^2
Inertia moment	J	1	kgm^2
Vertical force	F_z	2206.5	N
Gain brake calliper	K_{b_cal}	10	Nm/bar
Time constant brake calliper	τ_{cal}	0.1	s
Time constant EHB	τ_{EHB}	0.1	s

Due to the non-linear characteristic of the system it is very sensitive the choice of the operating points, so, in order to better choose them it was analyzed the system regarding the maximum values where it would remain stable. Those values are shown in table VI.

TABLE VI
Maximum parameters values to guarantee stability.

Type of road	λ_{ma}	μ_{max}	T_{be}	$\dot{\mu}_{max}$
Dry tarmac	0.17	1.17	$806.2380Nm$	30.1896
Wet tarmac	0.13	0.80134	$553.2324Nm$	28.6385
Snow	0.06	0.19004	$131.6348Nm$	18.2529
Ice	0.13	0.05	$34.6793Nm$	15.3195

The operating points were chosen in a way not to pass the maximum points in table VI but to get as close as possible to

TABLE VII
SYSTEM OPERATING POINT.

Vehicle speed	$50km/h$
Wheel speed	$50km/h$
Slip	0.057
Braking torque	493.6446 Nm
Friction coefficient	0.7126

the maximum friction coefficient. The operating points used for linearizing the system are listed in table VII.

For tuning, the system was simulated in closed-loop, considering simplified Burckhardt tire-road interection model, using Ziegler-Nichols tuning method explored in previous section. The obtained critical gain and period were $K_{cr} = 399$ and $T_{cr} = 0.35$ what result in the PID controller gains: $K_P = 113$, $K_I = 752.4$ and $K_D = 15.517$.

The modeled system was simulated in open-loop and closed-loop with PID controller for the different tire-road interaction model considering wet road condition. The simulation results are presented in figures from 6 to 11. The vehicle speed is represented by the blue curve and the wheel speep by the red one.

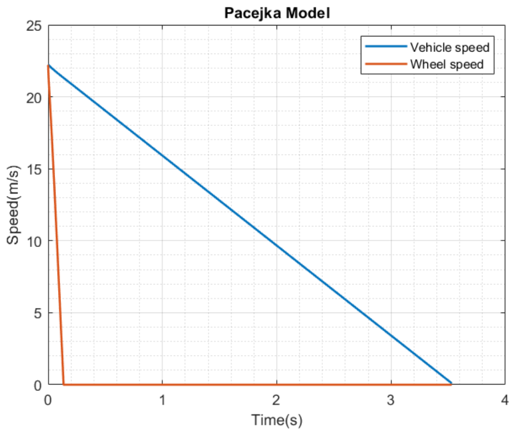

Fig. 6. Vehicle speed vs wheel speed while braking without ABS considering Pacejka model.

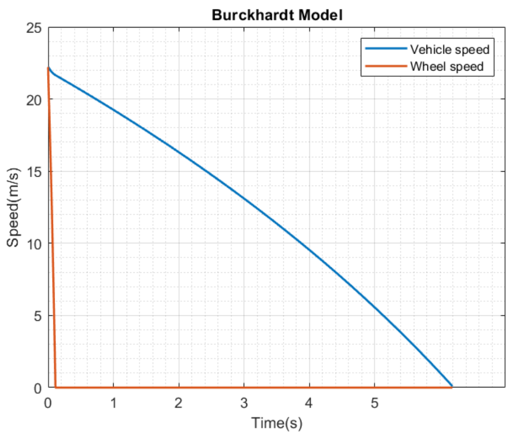

Fig. 7. Vehicle speed vs wheel speed while braking without ABS considering Burckhardt model.

For the system without ABS it is possible to see that disregard the tire-road interaction model, the wheel gets to a lockage status in less than thirty seconds of vehicle braking what means that during the remaining time until vehicle full stop, the driver does not detain control over the vehicle.

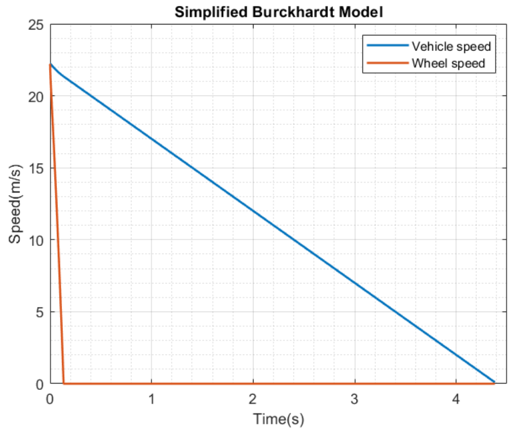

Fig. 8. Vehicle speed vs wheel speed while braking without ABS considering simplified Burckhardt model.

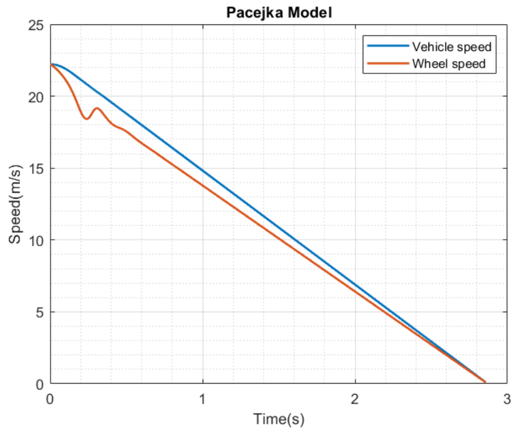

Fig. 9. Vehicle speed vs wheel speed while braking with ABS considering Pacejka model.

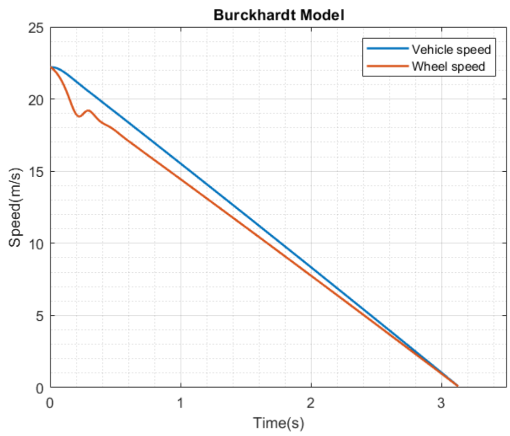

Fig. 10. Vehicle speed vs wheel speed while braking with ABS considering Burckhardt model

Once the ABS system is considered, the wheel deceleration decayed together with the vehicle deceleration. It is possible to note also that the necessary time until the vehicle perform a full stop got reduced.

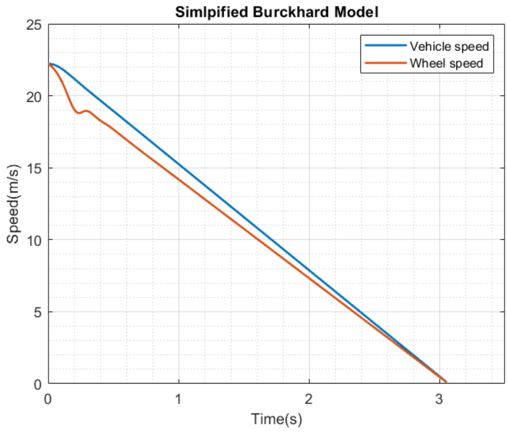

Fig. 11. Vehicle speed vs wheel speed while braking with ABS considering simplified Burckhardt model.

From the simulation graphs it is possible to conclude that the three models presented similar results, mainly in the controlled scenario.

V. CONCLUSIONS

In this paper different models of tire-road interaction were simulated for an ABS braking system. It was performed simulations both open loop and closed loop to verify the differences between the used models and efficiency of the projected controller.

It was projected a PID controller using modified Ziegler-Nichols method of critical gain. The controller was projected considering the simplified Burckhardt interaction model and applied for the Pacejka and Burckhardt representations.

It was possible to verify that the projected controller worked similarly for all three models, keeping wheel deceleration close to vehicle deceleration and reducing stopping distance if compared to the model simulated without the ABS system implemented. This way, applications of model-switching between those tire-road representations when developing ABS project, design and verification would not compromise the work.

The PID control, although, does not allow to treat constraints. Considering that usually vehicular systems are characterized by restrictions that can be of logic nature, regulations or mechanical nature, a control based on model of predictive type seems to be a better solution.

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