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Done CI9-EE-13 Hydroinformatics -...

Reassessment coursework (Hydroinformatics CI9-EE-13)

Please solve ALL the following questions.

Organize all your files in a readable manner, compress them and submit as a single (.zip) file.

Marking criteria: Code correctness, Code efficiency, Code Readability

Expected Time to complete: 10 hours

Problem 1 (40 Marks) – Expected Time to Complete (4 hours)

Relevant course material: Matrix/vector operations, looping structures, functions

One of the most important systems of partial differential equations (PDEs) in physics, chemistry and biology corresponds to the following reaction diffusion system

$$\frac{du}{dt} = r_u \nabla^2 u - uv^2 + f(1-u)$$

$$\frac{dv}{dt} = r_v \nabla^2 v + uv^2 - (f+k)v$$

This system of PDEs describes the time evolution of the concentration of 2 substances (u, v) in space and can be solved on a regular square grid with $\delta x = \delta y = h$ using the following discretization.

$$u_{i,j}^{(n)} = u_{i,j}^{(n-1)} + \delta t \left[\frac{u_{i-1,j}^{(n-1)} + u_{i+1,j}^{(n-1)} + u_{i,j-1}^{(n-1)} + u_{i,j+1}^{(n-1)} - 4u_{i,j}^{(n-1)}}{h^2} - u_{i,j}^{(n-1)} \left(v_{i,j}^{(n-1)}\right)^2 + f\left(1 - u_{i,j}^{(n-1)}\right) \right]$$

and

$$v_{i,j}^{(n)} = v_{i,j}^{(n-1)} + \delta t \left[\frac{v_{i-1,j}^{(n-1)} + v_{i+1,j}^{(n-1)} + v_{i,j-1}^{(n-1)} + v_{i,j+1}^{(n-1)} - 4v_{i,j}^{(n-1)}}{h^2} + u_{i,j}^{(n-1)} \left(v_{i,j}^{(n-1)}\right)^2 + (f+k) v_{i,j}^{(n-1)} \right]$$

(n) is the $n-th$ time step of the algorithm that evolves with time increments δt, and (i, j) are the spatial indices of the $i-th$ and $j-th$ element on the regular grid.

Develop the Matlab code to solve this system of coupled PDEs using the above discretization for the regular grid

$x = [0,1,2 \dots ,200]$
$y = [0,1,2, \dots 200]$

An initial condition

$u = 1$ for all x, y

$$v = \begin{cases} 1 \text{ for } 30 < x < 40 \text{ and } 50 < y \le 60 \\ 1 \text{ for } 40 < x < 50 \text{ and } 60 < y \le 70 \\ 0, \text{elsewhere} \end{cases}$$

For $t \in [0\ 1000]$

Investigate the cases

a) $f = 0.055;\ k = 0.062;\ r_u = 1; r_v = 0.5$
b) $f = 0.018;\ k = 0.051;\ r_u = 1; r_v = 0.5$

the regular grid

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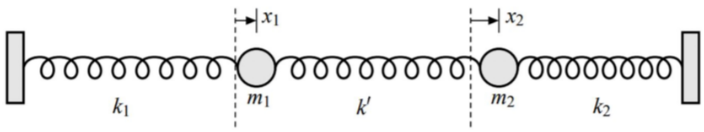
For all cases use periodic boundary conditions (https://en.wikipedia.org/wiki/Periodic_boundary_conditions). By trial and error choose appropriate δt values to avoid numerical instabilities.

Minimize as possible the use of looping structures for computational efficiency.

Problem 2 (40 Marks) – Expected Time to Complete (4 hours)

Relevant course material: Matrix/vector operations, ODEs, functions



The kinematics of the system of 2 masses and 3 springs shown in the Figure above are described by the following system of second order ordinary differential equations

$$\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} = -\begin{pmatrix} k_1 + k' & -k' \\ -k' & k_2 + k' \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

where $\ddot{x}_1, \ddot{x}_2$ are the accelerations (i.e. 2nd time derivatives) of the masses m_1, m_2 respectively, k_1, k', k_2 are the spring constants and x_1, x_2 the displacement of the masses from their equilibrium point. Develop the Matlab code that will solve numerically this system using Matlab's built in functionalities (i.e. **ode45**). Your program should be a function that accepts as arguments the masses (m_1, m_2), the spring constants (k_1, k_2, k'), the initial displacements and velocities $(x_1(t=0), x_2(t=0), \dot{x}_1(t=0), \dot{x}_2(t=0)\ )$ and the time of integration T. The outputs of the function will be a [2×N] matrix containing the displacements of the masses and a [1×N] vector containing the respective time for the displacements.

Problem 3 (20 Marks) – Expected Time to Complete (2 hours)

Relevant course material: functions, minimization

Develop the Matlab code to perform both constraint and non-constraint minimization of the following functions:

a) $f(x,y) = (x + 2y - 7)^2 + (2x + y - 5)^2, x \in [-10,10], y \in [-10\ 10]$
b) $f(x,y) = 100\sqrt{|y - 0.01x^2|} + 0.01|x + 10|, x \in [-15, -5], y \in [-3,3]$
c) $f(x,y) = 0.26(x^2 + y^2) - 0.48xy, x \in [-10,10], y \in [-10\ 10]$
d) $f(x,y) = (x^2 + y - 11)^2 + (x + y^2 - 7)^2, x \in [-5,5], y \in [-5\ 5]$

The function limits apply only for the constrained minimization